Beam based calibration for beam position monitor

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Contents

• Calibration at the beginning
  – Including mapping of BPM head, alignment, and gain calibration of electric circuit.

• Beam based alignment
  – BBA in next year beam commissioning of KEKB.

• Beam based gain calibration
  – Gain calibration of BPM in the KEKB
    • We have done BBGC in every 2-3 month since 2003.
  – New method for gain calibration of BPM in the J-PARC
    • Total least squares method is applied in J-PARC

• High-precision BPM system
  – Monitoring the consistency error and applying the beam-based gain calibration
Calibration of BPM

**Systematic errors in BPM**

- Setting error of BPM to Q-mag.
  Measurement of $\Delta x = ?, \Delta y = ?$
  
- Offset error

- Imbalance error in 4-outputs
  Relation among $V_1, V_2, V_3, V_4$
  when the beam locates in the center

**Gain error**

- the output signals travel through 4 separate cables, connectors, attenuators, and switches
STEP 1, Mapping of BPM head

All BPMs were mapped at a test bench with a movable antenna.

Test bench for the mapping

![Test bench image]

Result for mapping

![Graphs showing test bench mappings of LER94φ and HERracetrack]
STEP 2, Alignment of BPM heads against to the Q-magnets

Measurement of the mechanical offsets of the BPM heads to the Q-magnets.

Photograph of Measurement tool

Results of alignment of BPM heads against to the reference plane of the Q-Magnets
STEP-3, Gain calibration of the electronics

Attenuation of cable, switch, electronics, etc.

We measured the distribution of signal attenuation of all the electronics. We used a dummy head instead of BPM heads.

Results for the ratio between output signal B, C, and D against A in all electronics.

Output signals: A, B, C, D
Beam Based Alignment

- Measurement of the offset of BPM to the field center of the adjacent Q-magnet using the beam.

Beam Based Gain Calibration

- Calibration of the gain imbalance among four outputs of a BPM using the beam.
Beam based alignment (BBA) - Principal -

• BBA is searching the a beam orbit which is insensitive to the change of field strength of Q-magnet.

• This orbit must passing through the magnetic center of Q-magnet.

• The measured beam position for this orbit is corresponding to offset origin of BPM.

\[ \Delta X_m = 0 \] then \( \delta x_m \) offset
Actual procedure for BBA

Bump orbits: -8, 0, 8 [mm]
$I_Q$ : -4, -2, 0, 2, 4 [A]
- current of correction coil
from figure (a),(b),(c),
$\Delta x/\Delta I_Q$ and $x (I_Q=0)$,
where $x=x_m$

$\Delta x / \Delta I_Q = 0$

$X=0.4913$ [mm]

The orbital change due to $\Delta I_Q$ can be monitored not only the BPM but by any other BPMs in the ring.
Distribution of offsets with BBA in J-PARC MR
Distributions of offsets with BBA
In KEKB

OFFSET by BBA Θ LER

After gain correction, offset by BBA Θ LER

Offset by BBA Θ HER

After gain correction, offset by BBA Θ HER
The BPM offsets measured by BBA were set in the data base for BPM system.
The effect of BBA correction in J-Parc MR

COD without / with the BBA offset data (red / blue lines)
The effect of BBA correction in KEKB MR
The relative gain of the output data may drift due to unpredictable imbalance among output signals from the pickup electrodes.

The output signals (Vi) must travel through separate paths, such as feed-through, cables, connectors, attenuators, switches, and then are measured by the signal detectors.

The most probable source of the gain drift is the change in the electrical characteristics of the transmission line of the signal by temperature fluctuation.

Such as reason, the calibration of the gains of every BPMs are need.
Gain conception of BPM model
The output voltage model \( (V_i) \) for beam based calibration

\[
V_i = qg_i F_i(x, y)
\]

- \(i\)-th electrode \((i=1,2,3,4)\)
- \((x,y)\): beam position
- \(q\): proportional factor to the beam current
- \(g_i\): relative gain factor
- \(F_i(x,y)\): response function normalized to \(F_i(0,0)=1\)
Response function of 4 outputs

\[ F_1(x, y) = 1 + a_1 x + b_1 y + a_2 (x^2 - y^2) + b_2 (2xy) + a_3 (x^3 - 3x^2 y) + b_3 (3xy^2 - y^3) + a_4 (x^4 - 6x^2 y^2 + y^4) + b_4 (x^3 y - xy^3) \]

\[ F_2(x, y) = F_1(-x, y), \quad F_3(x, y) = F_1(-x, -y), \quad F_4(x, y) = F_1(x, -y) \]

These coefficients \((a_1, \ldots, a_4), (b_1, \ldots, b_4)\) were determined by fitting the calculated mapping by the finite boundary element method.
Beam mapping & Gain analysis

Measurement of m times beam positions by changing beam orbit.

\[ V_{i,j} = g_i q_j F_i(x_j, y_j) \]

\[ i = 1, 4, j = 1, m \]

Analysis by non-linear least square method

\[ J(a) = \sum_{i=1}^{4} \sum_{j=1}^{m} [V_{i,j} - g_i q_j F_i(x_j, y_j)]^2 \]

\[ a = (g_2, g_3, g_4, q_1, x_1, y_1, \ldots, q_m, x_m, y_m) \quad \text{fitting parameters} \]

but \[ g_1 = 1, g_2 = g_2 / g_1, g_3 = g_3 / g_1, g_4 = g_4 / g_1 \]

To minimize the sum of the squares of residuals, the software technique is a Marquardt method.
Mapping data by steering magnets for the gain calibration

LER_QC2RP

HER_QC2RE

m=25
Result of calibration in KEKB

These data are first measurements by beam based gain calibration

The BPM gains measured by BBGC were also set in the data base for BPM system
Correlation in the offset between BBA and BBGC

\[
x_{\text{offset}} = K \cdot \frac{1 - 1/g_2 - 1/g_3 + 1/g_4}{1 + 1/g_2 + 1/g_3 + 1/g_4}
\]

\[
y_{\text{offset}} = K \cdot \frac{1 + 1/g_2 - 1/g_3 - 1/g_4}{1 + 1/g_2 + 1/g_3 + 1/g_4}
\]

K is coefficient to position

We converted the measured value of BPM gain into BPM offset value, and tried to compare the offset with the offset from BBA.
Correlation in the offset between BBA and BBGC

Some part of BBA is caused by gain drift.
BPM head with diagonal cut in J-PARC MR

The horizontal and vertical beam positions are independently detected by two pairs of pickup
Output of diagonal cut electrodes

\[ V_L = \lambda (1 + \frac{x}{a}), \quad V_R = g_R \lambda (1 - \frac{x}{a}), \]
\[ V_D = g_D \lambda (1 - \frac{y}{a}), \quad V_U = g_U \lambda (1 - \frac{y}{a}), \]

\( \lambda \): proportional factor to beam current  
\( a \): radius of diagonal cut cylinder  
\( g_R, g_U, g_D \): relative gain to the \( g_L \)  
\( x, y \): beam position

By eliminate \( \lambda, x, y \) and \( a \)

\[ V_L = -\frac{V_R}{g_R} + \frac{V_U}{g_D} + \frac{V_D}{g_D} \]
Expression of m times measurement

Matrix representation

\[ A x = b \]

\[ A = \begin{pmatrix} -V_{R,1} & V_{U,1} & V_{D,1} \\ \vdots & \vdots & \vdots \\ -V_{R,m} & V_{U,m} & V_{D,m} \end{pmatrix}, \quad x = \begin{pmatrix} 1/g_R \\ \vdots \\ 1/g_D \end{pmatrix}, \quad b = \begin{pmatrix} V_{L,1} \\ \vdots \\ V_{L,m} \end{pmatrix}, \quad g_L = 1 \]
What LS & TLS

Reference: I. Markovsky and S. V. Huffel, Signal Processing 87

\[
\Delta r = \sum_{i=1}^{m} \left(-g_R V_{R,i} + g_U V_{U,i} + g_D V_{D,i} - V_{L,i}\right)^2
\]

\[
V_R, V_U, V_D \text{ have no error }, V_L \text{ have error}
\]

\[
\Delta d = \frac{1}{\|\mathbf{G}_\perp\|^2} \sum_{i=1}^{m} \left(-g_R V_{R,i} + g_U V_{U,i} + g_D V_{D,i} - V_{L,i}\right)^2
\]

\[
\mathbf{G}_\perp = (-1, -g_R, g_U, g_D)
\]

Normal vector to the plane in \((V_R, V_U, V_D, V_L)\) space expressed as

\[
-g_R V_R + g_U V_U + g_D V_D - V_L = 0
\]
Linear least squares

**Least squares**

Put the $A^T$ to both sides

$$A^T Ax = A^T b$$

Put the $[A^T A]^{-1}$ to both sides

$$[A^T A]^{-1} A^T Ax = [A^T A]^{-1} A^T b$$

\[ \downarrow \]

$$x_{LS} = (A^T A)^{-1} A^T b$$

$A$ have no error, $b$ have error

**Total least squares**

$$A^T Ax = A^T b$$

$$[A^T A - \sigma^2 I] x = A^T b$$

Put the $[A^T A - \sigma^2 I]^{-1}$ to both sides

$$x_{TLS} = \left[ (A^T A)^{-1} - \sigma^2 I \right]^{-1} A^T b$$

$\sigma$: the smallest singular value of $[Ab]$  

$I$: unit matrix  

$A^T$: transposed matrix of $A$

$A$ and $b$ have error
Simulation of LS & TLS

For the simulation, the gains set $g_R = 1.01, g_U = 1.005, g_D = 0.975$

$$V_L = \lambda \left(1 + \frac{x}{a}\right), \quad V_R = g_R \lambda \left(1 - \frac{x}{a}\right),$$

$$V_D = g_D \lambda \left(1 - \frac{y}{a}\right), \quad V_U = g_U \lambda \left(1 - \frac{y}{a}\right),$$

5x5 = 25 positions with 0.2% Gaussian noise. => 12500 points

**Simulation result**

<table>
<thead>
<tr>
<th></th>
<th>$g_R$</th>
<th>$g_U$</th>
<th>$g_D$</th>
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<tbody>
<tr>
<td>True</td>
<td>1.01</td>
<td>1.005</td>
<td>0.975</td>
</tr>
<tr>
<td>LS</td>
<td>1.034</td>
<td>1.015</td>
<td>0.988</td>
</tr>
<tr>
<td>TLS</td>
<td>1.012</td>
<td>1.005</td>
<td>0.977</td>
</tr>
</tbody>
</table>

Reconstructed mapping data. Black is Position $(x,y)$ without correction. Red is $(x,y)$ with TLS correction.

TLS method indicates good reproducibility of gains.
Beam test
Output data in nine displacements of beam positions at a BPM.

Reconstructed mapping data.
Red: \((x, y)\) without correction,
Black: \((x, y)\) with TLS gains

\[ \Delta g \approx 0.5\% \]
Difference in the relative gains between LS and TLS

<table>
<thead>
<tr>
<th></th>
<th>(g_L)</th>
<th>(g_U)</th>
<th>(g_D)</th>
</tr>
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<tbody>
<tr>
<td>BPM001</td>
<td>TLS</td>
<td>1.0062</td>
<td>1.0024</td>
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<tr>
<td></td>
<td>LS</td>
<td>1.0103</td>
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<tr>
<td>BPM002</td>
<td>TLS</td>
<td>0.9568</td>
<td>0.9811</td>
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<tr>
<td></td>
<td>LS</td>
<td>0.9617</td>
<td>0.9838</td>
</tr>
</tbody>
</table>
Result of the relative gains of all BPMs in J-Parc MR

Kick angles of the steering magnets

To obtain the data with mapped beam positions

Blue: Low intensity as $10^{13}$[PPP]
Red: High intensity as $10^{14}$[PPP]
Evaluation of gain correction

4 buttons pickups at the KEKB

- Beam position calculation from the 4 outputs

\[
X = \frac{V_1 - V_2 - V_3 + V_4}{V_1 + V_2 + V_3 + V_4}, \quad Y = \frac{V_1 + V_2 - V_3 - V_4}{V_1 + V_2 + V_3 + V_4}
\]

\[
x = F_X(X,Y), \quad y = F_Y(X,Y)
\]

\[
F_X(X,Y) = a_0 + a_1 X + a_2 Y + a_3 X^2 + a_4 XY + a_5 Y^2 \\
+ a_6 X^3 + a_7 X^2 Y + a_8 XY^2 + a_9 Y^3
\]

\[
F_Y(X,Y) = b_0 + b_1 X + b_2 Y + b_3 X^2 + b_4 XY + b_5 Y^2 \\
+ b_6 X^3 + b_7 X^2 Y + b_8 XY^2 + b_9 Y^3
\]

The coefficients \((a_n, b_n)\) are obtained by fitting of the mapping data.
Four Positions by selected 3 outputs

The beam position is also obtainable from the output voltage of any three electrodes chosen out of four electrodes.

\[
\begin{align*}
X_1 &= (V_1 - V_2)/(V_1 + V_2), \quad X_2 = (V_3 - V_4)/(V_3 + V_4) \\
Y_1 &= (V_2 - V_3)/(V_2 + V_3), \quad Y_2 = (V_1 - V)/(V_1 + V)
\end{align*}
\]

Normalization

\[
\begin{align*}
(x_1, y_1) &= \left( F_X^{ABC}(X_1, Y_1), \quad F_Y^{ABC}(X_1, Y_1) \right) \\
(x_2, y_2) &= \left( F_X^{BCD}(X_2, Y_1), \quad F_Y^{BCD}(X_2, Y_1) \right) \\
(x_3, y_3) &= \left( F_X^{ACD}(X_2, Y_2), \quad F_Y^{ACD}(X_2, Y_2) \right) \\
(x_4, y_4) &= \left( F_X^{ABD}(X_1, Y_2), \quad F_Y^{ABD}(X_1, Y_2) \right)
\end{align*}
\]

3rd order polynomials

\[
\begin{align*}
\sigma_X &= \sqrt{\frac{1}{4} \sum_{i=1}^{4} (X_i - \bar{X})^2} \quad \text{with} \quad \bar{X} = \frac{1}{4} \sum_{i=1}^{4} (X_i) \\
\sigma_Y &= \sqrt{\frac{1}{4} \sum_{i=1}^{4} (Y_i - \bar{Y})^2} \quad \text{with} \quad \bar{Y} = \frac{1}{4} \sum_{i=1}^{4} (Y_i)
\end{align*}
\]

Consistency error

If the four outputs have ideal correlation, \(((x_1, y_1), (x_2, y_2), (x_3, y_3), (x_4, y_4))\) should coincide with each other.
Consistency error in LER at KEKB

Before gain calibration

After gain calibration

The consistency error became very small
Four positions of diagonal cut pickups at J-PARC

\[ x_1 = \frac{V_L - V_R / g_R}{V_L + V_R / g_R} a, \quad y_1 = \frac{V_U / g_U - V_D / g_D}{V_U / g_U + V_D / g_D} a, \]

\[ x_2 = \frac{V_L - V_R / g_R}{V_U / g_U + V_D / g_D} a, \quad y_2 = \frac{V_U / g_U - V_D / g_D}{V_L + V_R / g_R} a, \]

\[ x_3 = \left( \frac{2V_L}{V_U / g_U + V_D / g_D} - 1 \right) a, \quad y_3 = \left( \frac{2V_U / g_U}{V_L + V_R / g_R} - 1 \right) a \]

\[ x_4 = \left( \frac{-2V_R / g_R}{V_U / g_U + V_D / g_D} + 1 \right) a, \quad y_4 = \left( \frac{-2V_D / g_D}{V_L + V_R / g_R} + 1 \right) a \]

\[ \left\{ \begin{array}{l}
\sigma_X = \sqrt{\frac{1}{4} \sum_{i=1}^{4} (X_i - \bar{X})^2} \quad \text{with} \quad \bar{X} = \frac{1}{4} \sum_{i=1}^{4} (X_i) \\
\sigma_Y = \sqrt{\frac{1}{4} \sum_{i=1}^{4} (Y_i - \bar{Y})^2} \quad \text{with} \quad \bar{Y} = \frac{1}{4} \sum_{i=1}^{4} (Y_i) 
\end{array} \right. \]

Consistency error
Consistency error of the beam test at J-PARC

<table>
<thead>
<tr>
<th></th>
<th>Before [mm]</th>
<th>Before [mm]</th>
<th>After [mm]</th>
<th>After [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>BPM001</td>
<td>0.524</td>
<td>0.518</td>
<td>0.018</td>
<td>0.018</td>
</tr>
<tr>
<td>BPM002</td>
<td>0.964</td>
<td>0.954</td>
<td>0.025</td>
<td>0.025</td>
</tr>
</tbody>
</table>

Consistency error before and after gain calibration of BPMs at J-PARC MR
Conclusions

• We should pay special attention to guarantee the precise measurement of beam positions over a long time.

• The BBA calibration is necessary for correction of the BPM offset error.

• The BBGC is also important for correction of the imbalanced gains among four outputs of a BPM.

We have realized a high-precision BPM system by monitoring the consistency error and applying the beam-based gain calibration.
Thank you for your attention
Diurnal variation of BPM consistency

Increasing the temperature made impedance of coaxial cable drift.

- BPM signal cables were installed from ground level to tunnel.
- Part of the total cable length are cabled in outdoor area.
- Effect of air temperature and sunshine caused the diurnal variation of BPM consistency.
- To avoid sunshine, cables are wrapped by the insulating sheet.